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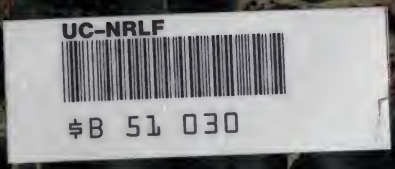

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A METHOD OF PETROGRAPHIC ANALYSIS

BASED UPON

Chromatic Interference with Thin Sections of Doubly-Refracting Crystals in Parallel Polarized Light.

Thesis

Presented to the Faculty of Philosophy of the University of Pennsylvania in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

BY

HOMER MUNRO DERR, A. M.

Hector Tyndale Fellow.

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Plate 1.

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PLATE 1.

Thin Section showing Free Gold in Metamorphic Diorite

FROM ENCAMPMENT, WYOMING.

Magnified 50 diameters:—*f*, feldspar, (oligoclase, albite); *q*, quartz; *h*, hornblende; *m*, magnetite; *g*, gold.
Crossed nicols; thickness of section, 56 microns.

h
q
f
q
f
m
h
g
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BEHAVIOR OF ISOMETRIC CRYSTALS.

Isometric crystals on account of their isotropic character exhibit no special phenomena in polarized light. If a thin plate of such a mineral be placed in the path of a polarized beam of light between the polarizer and the analyzer, the beam will experience no alteration of its plane of vibration, no matter in what direction the plate was cut from the crystal, nor in what position it lies between the polarizer and the analyzer; for the elasticity of the ether is the same in all directions in such a case, and rotating the mineral about any axis whatever will effect no change of the plane of vibration of the polarized light. If the section is colorless, it will not alter the color or the brightness of the field of view in a polarization microscope, except for the small amount of absorption which a ray of light will experience in passing through any medium. If the section is colored, the field of view will appear slightly different in color from that of the mineral; however, this color does not change by altering the position of the plate in any way. Furthermore, when the nicols are crossed, a section on the stage of the microscope appears dark, and revolving it in any plane produces no effect. It appears light, and experiences no change in any position between parallel nicols. Sections of transparent isometric crystals may always be recognized as such by the fact that they behave as an amorphous substance in polarized light. There are, however, some optical anomalies in this connection. Sections of amorphous or isometric minerals sometimes appear light when viewed between crossed nicols. These results are due to internal strains caused either by inclusions of gases or fluids which exert a pressure on their surroundings, or by contraction of adjacent parts. These phenomena may usually be distinguished from ordinary double refraction, since the appearance is not generally uniform throughout the section.

INTERFERENCE.

Fresnel and Arago investigated completely the conditions of interference of two rays polarized at right angles to each other

after they had been brought back to the same plane of polarization by passing them through a crystal of calc-spar whose principal section made an angle of 45° with the planes of polarization of each of the two rays. They found, (1) that two rays polarized at right angles to each other, which have come from an unpolarized ray, do not interfere even when they are brought into the same plane of polarization; (2) that two rays polarized at right angles, which have come from a polarized ray, interfere when they are brought back to the same plane of polarization.

Let a ray of polarized monochromatic light be incident at the lower surface of a plane-parallel doubly-refracting plate at any angle, Fig. 1, at the point A. It is separated into two rays AD and AE, vibrating in planes at right angles to each other and following different paths in the plate. On emergence they follow paths parallel to their direction at incidence, but are not coincident, and do not produce interference. But other parallel rays from the same source and incident at B and C, will emerge from the upper surface, so that from all points E and F, the ordinary component of one ray and the extraordinary of another follow the same path. These rays will have travelled over slightly different paths in the plate, and with different velocities; then on emergence one ray must have advanced a certain number of wave-lengths, or fractions thereof, ahead of the other. The waves are therefore in different phases and retain this difference of phase while they travel in the same medium. The plane of vibration of the extraordinary ray is parallel to the principal optic section of the plate, and that of the ordinary ray is at right angles to the same. If an analyzer be placed in the path of these rays, each ray will be by it resolved into components the vibrations of which are in and at right angles to the principal section of the polarizer, and only

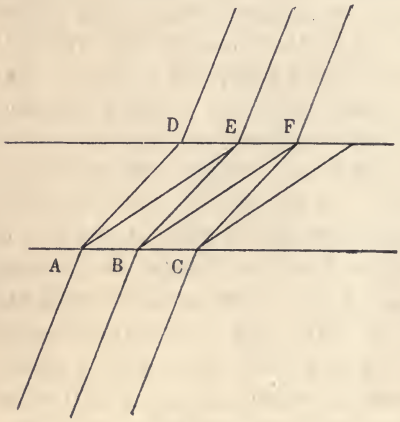

FIG. 1.

the former will be transmitted. That is to say, there will emerge two rays advancing in the same line and with parallel vibrations; hence they are capable of interfering, since light-rays can completely interfere only when their vibration are in the same plane. If these vibrations are in the same phase, the intensity of the resultant wave will be proportional to the square of the sum of their amplitudes; but if in opposite phases, the intensity will be proportional to the square of the difference.

WITH PARALLEL MONOCHROMATIC LIGHT AND CROSSED NICOLS.

In sections cut perpendicular to the optic axis, interference phenomena are impossible, and the field remains dark throughout a rotation of 360°, since the light from the polarizer traverses the section in the direction of the optic axis, and therefore without change. In all other sections there is double refraction, and consequently interference. The field is dark four times during the entire rotation of the stage at intervals of 90°; that is, when the planes of vibration of the rays produced in the section coincide with the principal sections of either nicol. For all other positions the field is illuminated by the components of the rays which penetrate the analyzer, and this brightening is most intense in the diagonal positions.

The rays pursuing the same path are brought into one plane of vibration by the analyzer and there interfere. The kind of interference is determined by the formula,

$$\triangle = h\left[n_1 - n_2 + \sin^2 i\left(\frac{1}{n_2} - \frac{1}{n_1}\right)\right],$$

in which $\triangle$ is the difference in the retardations in microns which the two rays have undergone.

h is the thickness of the section in microns.
λ is the wave-length in microns.
i is the angle of incidence.
n_1 is the index of refraction for the slower ray.
n_2 is the index of refraction for the faster ray.

For normal incidence, which is usually the case in practice, $\triangle = h\,(n_1 - n_2)$.

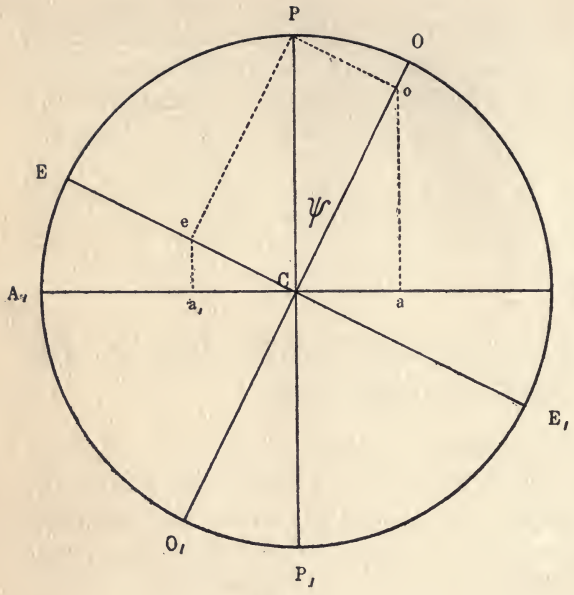

FIG. 2.

When $\triangle = \lambda$, 2λ, 3λ, etc., the field is dark during an entire revolution of the section on the stage. From Fig. 2, it will be seen that the components of PP_1, on emergence from the plate with vibration directions OO_1 and EE_1, must be of the same phase; that is, the simultaneously displacing forces acting upon any ether particle C are Co and Ce, which when reduced to the plane of the analyzer are Ca and Ca_1, equal and in opposite directions.

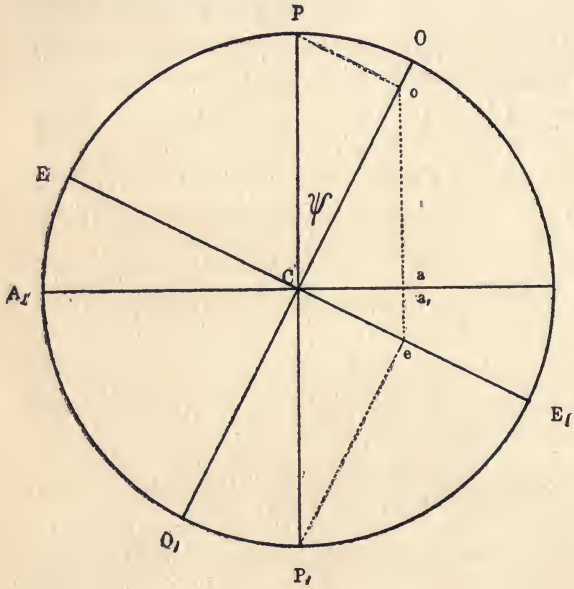

FIG. 3.

When $\triangle = \frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc., the light will be at its brightest because the components of PP_1 must then be of opposite phase on emergence from the plate. From Fig. 3, the simultaneously displacing forces acting on any ether particle C are Co and Ce, which reduced by the

analyzer to its plane are Ca and Ca_1, equal and in the same direction.

Let a thin wedge of doubly-refracting crystal, such as quartz, be cut so that its planes of vibration are parallel to the length and breadth, and then examined between crossed nicols, using perpendicularly incident monochromatic light. Upon rotation, it will appear dark when in the "normal positions," but in all others it will show a series of dark and light bands which are most marked in the diagonal position. When the nicols are made parallel the portions formerly light become dark, and vice versa. The distance between the dark bands varies with light of different wave-lengths.

The formula for intensity of the emerging light is

$$I = a^2 \sin^2 2\Psi \sin^2 \left[\frac{\pi \Delta}{\lambda} \right],^*$$

in which a = the amplitude of the incident wave, λ = the wave-length, Δ = the retardation, Ψ = the angle between the vibration plane of the polarizer and the slower wave.

I will be a minimum:

(*a*) when $\sin^2 2\Psi = 0$, or $2\Psi = 0°, 180°, 360°, 540°$, etc., or $\Psi = 0°, 90°, 180°, 270°$, etc.; that is, four times in a revolution of the section, or whenever the planes of vibration of the plate coincide with those of the polarizer and the analyzer.

(*b*) when $\sin^2 \left[\frac{\pi \Delta}{\lambda} \right] = 0$, which will be whenever $\frac{\Delta}{\lambda} = 1$, 2, 3, etc.; that is, whenever the phase difference Δ is a multiple of λ, for then $\sin^2 \left[\frac{\pi \Delta}{\lambda} \right]$ becomes $\sin^2 180°$, or a mutiple thereof. This is independent of Ψ; hence with this condition, the section will appear dark throughout an entire revolution.

I will be a maximum:

When $\sin^2 2\Psi = 1$ or $2\Psi = 90°, 270°$, etc., and $\Psi = 45°, 135°, 225°$, etc.

When $\frac{\Delta}{\lambda} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, etc., for then $\sin^2 \left[\frac{\pi \Delta}{\lambda} \right] = \sin^2 90°, \sin^2 270°$, etc., $= 1$.

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* See Derivation of Formulae, page 21.

WITH PARALLEL WHITE LIGHT AND CROSSED NICOLS.

In sections cut normal to the optic axis, the field appears dark throughout a rotation of 360°, the optic axis being the same for all colors. In all other sections there is extinction every 90°, and greatest brightness in the diagonal positions; but, since Δ may be at the same time approximately an even multiple of $\frac{1}{2}\lambda$ and an odd multiple of $\frac{1}{2}\lambda^1$, light of one wave-length may be greatly weakened while that of another wave-length is practically undimmed; that is, there will result a tint due to the partial extinction of some colors.

For example, suppose that in a polarization microscope, parallel white light passes upward through the polarizer, whose principal section is represented by PP_1 in Fig. 2. Next let this light, which is polarized in a single plane PP_1, pass through a thin section of gypsum. It will there be separated into two waves vibrating in planes at right angles to each other, OO_1 and EE_1. The two waves travel through the section with unequal velocity, and on emerging one is retarded a certain number of wave-lengths, or fractions thereof, as compared with the other. Now let these light waves pass through the analyzer, with its vibration-plane AA_1 at right angles to PP_1. Then each of the two sets of vibrations will have a component in the direction AA_1, and these will emerge polarized in the same plane, and are therefore capable of interfering. Light corresponding to the other components in the direction PP_1 will be extinguished. As previously stated, one of the emergent waves is slightly retarded as compared with the other. The amount of this retardation, on which the interference-color of the section depends, is proportional to the difference of the indices, and also to the thickness of the section.

If in the examination of the plate of gypsum above, one of the nicols had been rotated 90°,—that is to say, if the principal sections of the two nicols had been parallel,—interferenee would have taken place between the emerging rays; but the color resulting in each case would have been exactly *complementary* to that obtained at first when the nicols were crossed. For instance, in Fig 2, if $\Psi = 0$, the principal sections of the plate coincide with those of polarizer and analyzer, and complete extinction occurs; but if AA_1 be made parallel to PP_1, evidently the section would appear white.

If we examine in the same way a very thin and gradually-tapering wedge of some doubly-refracting crystal (quartz for example), cut so that its planes of vibration are parallel to the length and breadth, the successive interference-colors of the *first order*, beginning at the thin end, pass from an iron-gray through bluish-gray to white, yellow, and red; then follow violet, indigo, blue, green, yellow, orange, and red of the *second order;* then the similar but paler series of colors of the *third order*, and finally the very pale shades of green and red of the *fourth order*. Beyond this the colors are not very distinct, and white of a higher order finally results from the interference. A mineral of very strong double refraction, such as calcite, shows only the white of the higher orders unless extremely thin.

In general, a thin section of a doubly-refracting crystal, examined between crossed nicols, is not dark except when placed in certain definite positions. In any other position it does not completely extinguish the light, but its effect, in conjunction with the nicols, is partially to suppress the several components of the white light in different degrees, so that in the emergent beam these components are no longer in the proportions to give white light. In this way arise interference-tints, which may be definitely classed according to Newton's color-scale. (See page 15.) The several tints, though graduating into one another, are distinguished by names and divided into several orders. The precise position in the scale of a given tint observed between crossed nicols can be fixed by means of a quartz-wedge or other contrivance for compensating or neutralizing the effect of double refraction of the section.

The interference-tints given by a crystal section depend (1) on the amount of double refraction of the mineral ($n_1 - n_2$), which is a specific character; (2) on the direction of the section relatively to the ellipsoid of optic elasticity, the tint being highest for a section parallel to the greatest and least axes of the ellipsoid; (3) on the thickness of the section. The last two are disturbing factors, which must be eliminated before we can use the interference-tints as an index of the amount of double refraction of the crystal, and as a useful criterion in identifying the mineral. The fact that the interference-tints depend in part on the direction of the section through the crystal will give little difficulty in estimating approximately the amount of double refraction of the mineral. If several crystals of the

same mineral are contained in a rock-section, it is sufficient to have regard to the ones which give the highest interference-tints. Even a single crystal will in the majority of cases give tints not so far below those proper to the mineral as to occasion error, but the possibility of the section having an unfavorable direction must be borne in mind.

In making a determination, the manner of proceedure is briefly as follows: Select a crystal, revolve it on the stage of the microscope until it is in the position of maximum intensity (i. e., when the greatest and least axes of elasticity make an angle of about 45° with the principal sections of polarizer and analyzer); introduce the quartz-wedge between polarizer and analyzer (in slot provided for the purpose, also making an angle of 45° with principal sections of the two nicols), until a position of maximum darkness of the crystal is reached. If the interference-tint of the crystal cannot in this position, or when revolved through an angle of 90°, be reduced through the successive colors of the scale, in descending order, back to darkness, the section is an unfavorable one, i. e., it is not cut approxmiately parallel to the plane of the axes of greatest and least elasticity of the crystal. When a favorable section is found, after having fixed the quartz-wedge in the position giving maximum darkness to the crystal, first remove the section from the stage of the microscope. Now the quartz-wedge, as viewed through the microscope, will be of the same color and order in Newton's scale as that of the crystal-section under the same conditions. Carefully withdraw the wedge, at the same time noting the succession of tints in descending order. In this fashion, one can estimate the order of the interference-color in the scale; and by applying the formula $\triangle = h\ (n_1 - n_2)$, in connection with the tables given on pages 15 and 16, many of the commoner rock-forming minerals may be identified.

The following twenty-four examples will illustrate:

Rock Section.	Thickness in Microns.	Crystals.	Interference Color between Crossed Nicols.	Interference Color between Parallel Nicols.	Order of Color.
QUARTZ-PORPHYRY	26.5	Quartz	Greenish white	Brown	I
		Orthoclase	Gray	Brownish yellow	I
RHYOLITE	32.8	Quartz	Straw-yellow	Deep violet	I
		Orthoclase	Almost pure white	Light red	I
		Biotite	Light greenish gray	Grayish red	IV
		Hornblende	Light green	Carmine-red	II
GRANITE	23.6	Quartz	Gray	Brownish yellow	I
		Orthoclase	Bluish gray	Brownish white	I
		Plagioclase	Bluish gray	Brownish white	I
		Microcline	Bluish gray	Brownish white	I
		Biotite	Greenish yellow	Grayish blue	III
		Muscovite	Bright orange-red	Greenish blue	II
		Hornblende	Purple	Light green	II
		Augite	Reddish orange	Bluish green	I
		Hypersthene	Pale straw-yellow	Dark reddish brown	I
		Tourmaline	Brownish yellow	Gray-blue	I
		Zircon	Greenish yellow	Grayish blue	III
		Apatite	Lavender-gray	Yellowish white	I
PEGMATITE	21.4	Quartz	Bluish gray	Brownish yellow	I
		Feldspar	Grayish blue	Brownish white	I
		Biotite	Greenish blue	Flesh-colored	III
		Muscovite	Pure yellow	Indigo	II
		Tourmaline	Brownish yellow	Gray-blue	I
		Beryl	Grayish blue	Brownish white	I
TRACHYTE	27.2	Orthoclase	Gray	Brownish yellow	I
		Biotite	Dull purple	Dull sea-green	III
		Hornblende	Sky-blue	Orange	II
		Augite	Indigo	Golden yellow	II
PHONOLITE	33.1	Orthoclase	Yellowish white	Carmine-red	I
		Augite	Greenish blue	Brownish orange	II
		Elaeolite	Grayish blue	Brownish white	I
TINGUAITE	18.4	Orthoclase	Grayish blue	Brownish white	I
		Biotite	Dark violet-red	Green	II
		Hornblende	Bright yellow	Blue	I
		Augite	Brownish yellow	Gray-blue	I
		Leucite	Black	Bright white	I
		Elaeolite	Lavender-gray	Yellowish white	I
SYENITE	24.7	Orthoclase	Bluish gray	Brownish yellow	I
		Augite	Red	Pale green	I
		Hornblende	Indigo	Golden yellow	II
		Biotite	Flesh-colored	Sea-green	III
NEPHELINE-SYENITE	19.6	Orthoclase	Grayish blue	Brownish white	I
		Nepheline	Lavender-gray	Yellowish white	I
		Hornblende	Reddish orange	Bluish green	I
		Augite	Brownish yellow	Gray-blue	I
		Biotite	Indigo	Impure yellow	III
ANDESITE	25.3	Plagioclase	Bluish gray	Brownish yellow	I
		Augite	Deep red	Yellowish green	I
		Biotite	Carmine-red	Green	III
		Hornblende	Indigo	Golden yellow	II

Rock Section.	Thickness in Microns.	Crystals.	Interference Color between Crossed Nicols.	Interference Color between Parallel Nicols.	Order of Color.
DACITE	33.7	Quartz	Light yellow	Indigo	I
		Plagioclase	Yellowish white	Carmine-red	I
		Biotite	Whitish gray	Bluish gray	IV
		Augite	Green	Light carmine-red	II
		Hornblende	Light green	Purplish red	II
DIORITE	29.1	Plagioclase	Greenish white	Brown	I
		Augite	Sky-blue	Orange	II
		Biotite	Bluish green	Lilac	IV
		Hornblende	Greenish blue	Brownish orange	II
BASALT	38.9	Plagioclase	Light yellow	Indigo	I
		Augite	Greenish yellow	Violet	II
OLIVINE-BASALT	41.7	Plagioclase	Bright yellow	Blue	I
		Augite	Pure yellow	Indigo	II
		Olivine	Flesh colored	Sea-green	III
HORNBLENDE-BASALT	46.3	Plagioclase	Bright yellow	Blue	I
		Augite	Bright orange-red	Greenish blue	II
		Hornblende	Dark violet red	Green	II
NEPHELINE-BASALT	43.6	Nepheline	Gray	Brownish yellow	I
		Augite	Orange	Dark blue	II
LEUCITE-BASALT	45.9	Leucite	Iron-gray	White	I
		Augite	Bright orange-red	Greenish blue	II
MELILITE-BASALT	51.3	Melilite	Straw-yellow	Deep violet	I
		Augite	Light bluish violet	Yellowish green	III
DIABASE	49.2	Plagioclase	Brownish yellow	Gray-blue	I
		Augite	Dark violet red	Green	II
GABBRO	46.5	Plagioclase	Bright yellow	Blue	I
		Augite	Bright orange-red	Greenish blue	II
NORITE	41.3	Plagioclase	Bright yellow	Blue	I
		Hypersthene	Reddish orange	Bluish green	I
TROCTOLITE	55.4	Plagioclase	Brownish yellow	Gray-blue	I
		Olivine	Whitish gray	Bluish gray	IV
THERALITE	33.4	Augite	Greenish blue	Brownish orange	
		Biotite	Whitish gray	Bluish gray	
		Plagioclase	Yellowish white	Carmine-red	
		Nepheline	Grayish blue	Brownish white	
		Apatite	Grayish blue	Brownish white	
PERIDOTITE	29.8	Olivine	Dark violet-red	Green	II
		Augite	Sky-blue	Orange	II
		Diopside	Bright orange-red	Greenish blue	II
		Hornblende	Greenish blue	Brownish orange	II
		Biotite	Light green	Carmine	IV

COLOR-SCALE ACCORDING TO NEWTON.

△ in Microns.	Interference Color between Crossed Nicols.	Interference Color between Parallel Nicols.	
0.000	Black	Bright white	First Order
0.040	Iron-gray	White	
0.097	Lavender-gray	Yellowish white	
0.158	Grayish blue	Brownish white	
0.218	Clearer gray	Brownish yellow	
0.234	Greenish white	Brown	
0.259	Almost pure white	Light red	
0.267	Yellowish white	Carmine-red	
0.275	Pale straw-yellow	Dark reddish brown	
0.281	Straw-yellow	Deep violet	
0.306	Light yellow	Indigo	
0.332	Bright yellow	Blue	
0.430	Brownish yellow	Gray-blue	
0.505	Reddish orange	Bluish green	
0.536	Red	Pale green	
0.551	Deep red	Yellowish green	
0.565	Purple	Lighter green	Second Order
0.575	Violet	Greenish yellow	
0.589	Indigo	Golden yellow	
0.664	Sky-blue	Orange	
0.728	Greenish blue	Brownish orange	
0.747	Green	Light carmine-red	
0.826	Lighter green	Purplish red	
0.843	Yellowish green	Violet-purple	
0.866	Greenish yellow	Violet	
0.910	Pure yellow	Indigo	
0.948	Orange	Dark blue	
0.998	Bright orange-red	Greenish blue	
1.101	Dark violet-red	Green	
1.128	Light bluish violet	Yellowish green	Third Order.
1.151	Indigo	Impure yellow	
1.258	Greenish blue	Flesh-colored	
1.334	Sea-green	Brownish red	
1.376	Brilliant green	Violet	
1.426	Greenish yellow	Grayish blue	
1.495	Flesh-colored	Sea-green	
1.534	Carmine-red	Green	
1.621	Dull purple	Dull sea-green	
1.652	Violet-gray	Yellowish green	Fourth Order.
1.682	Grayish blue	Greenish yellow	
1.711	Dull sea-green	Yellowish gray	
1.744	Bluish green	Lilac	
1.811	Light green	Carmine	
1.927	Light greenish gray	Grayish red	
2.007	Whitish gray	Bluish gray	

DOUBLY-REFRACTING POWER OF VARIOUS MINERALS.

CRYSTAL.	$n_1 - n_2$	CRYSTAL.	$n_1 - n_2$
Proustite	0.2953	Phenacite	0.0157
Sulphur	0.2900	Anorthite	0.0130
Rutile	0.2871	Natrolite	0.0119
Dolomite	0.1791	Hyphersthene	0.0115
Calcite	0.1722	Andalusite	0.0110
Brookite	0.1580	Gypsum	0.0100
Aragonite	0.1558	Bronzite	0.0100
Cassiterite	0.0968	Corundum	0.0092
Zircon	0.0618	Quartz	0.0091
Biotite	0.0600	Topaz	0.0090
Epidote	0.0545	Cordierite	0.0090
Anhydrite	0.0430	Axinite	0.0090
Muscovite	0.0420	Orthoclase	0.0080
Olivine	0.0360	Albite	0.0080
Scapolite	0.0360	Beryl	0.0073
Diopside	0.0335	Plagioclase	0.0071
Cancrinite	0.0289	Zoisite	0.0060
Tremolite	0.0265	Melilite	0.0058
Hornblende	0.0240	Nephelite	0.0049
Augite	0.0220	Elaeolite	0.0047
Sillimanite	0.0220	Apatite	0.0042
Glaucophane	0.0216	Vesuvianite	0.0020
Brucite	0.0210	Apophyllite	0.0019
Tourmaline	0.0189	Leucite	0.0010
		Chlorite	0.0010

Plate 2.

PLATE 2. (Crossed Nicols.)

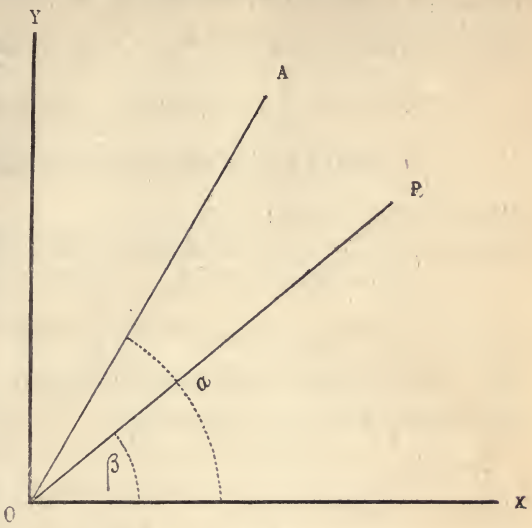
Chalcedony,

Section cut at right angles to the length of a stalactite ; × 100 diameters. Central sections through the concretions give a spherulitic interference cross. Radial fibers appear to be uniaxial, since extinction is complete in fibers parallel to planes of vibration of polarizer and analyzer, i. e., when $\Psi = 0$; the illumination is at a maximum when $\Psi = 45°$. Extinction is also complete where fibers are cut at right angles, for the light travels in the direction of their optic axes. The crystal system of this mineral has not yet been definitely determined ; however, it appears to be optically uniaxial.

DERR

DERIVATION OF FORMULAE.

Let there be a crystalline plate having OX and OY for axes of elasticity, OP the direction of a polarizer on this section (fig. 5) ; a ray of light polarized by OP vibrating in the principal section of the polarizer is divided, in the plate, into two rays, which may be regarded as superposed and which have at emergence a difference of phase :



$$\varphi = \frac{h}{\lambda \cos r} (n_e - n_o).$$

Let us call x and y the vibrations of these rays which give as a resultant an elliptical movement, and let us receive the corresponding doubly-refracted ray on an analyzer having OA for the direction on the plane of the plate ; the angle between the polarizer and the axis OX of the plate is POX. Put $\angle$ POX $= \beta$. The angle between the analyzer and the same axis is AOX, and we put $\angle$ AOX $= \alpha$. The components belonging to OX and OY are :

$$x = a \cos 2\pi \frac{t}{T} \cos \beta,$$

$$y = a \cos 2\pi \frac{t}{T} \sin \beta.$$

They give rise to two rays which emerge from the plate with a difference of phase φ ; the components of these rays are :

$$x = a \cos 2\pi \frac{t}{T} \cos \beta,\ y = a \cos 2\pi \left[\frac{t}{T} - \varphi \right] \sin \beta.$$

x gives to OA a component $\xi = x \cos \alpha$ parallel to the principal section and a perpendicular ; y gives to OA a component $\eta = y \sin \alpha$ parallel to its principal section and a perpendicular to that section. We need not concern ourselves with the perpendiculars, which are eliminated ; the parallels give a resultant, which is their algebraic sum :

$$\xi + \eta = x \cos \alpha + y \sin \alpha.$$

Replacing x and y by their values, we have

$$\xi + \eta = a \cos \alpha \cos \beta \cos 2\pi \frac{t}{T} + a \sin \alpha \sin \beta \cos 2\pi \left(\frac{t}{T} - \varphi \right)$$

$$= a \cos 2\pi \frac{t}{T} (\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos 2\pi \varphi)$$

$$+ a \sin 2\pi \frac{t}{T} \sin \alpha \sin \beta \sin 2\pi \varphi.$$

According to Fresnel, let us put

$$a (\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos 2\pi \varphi) = E \cos \Delta,$$

$$a \sin \alpha \sin \beta \sin 2\pi \varphi = E \sin \Delta.$$

$$\eta + \xi = \cos 2\pi \frac{t}{T} \cos \Delta + \sin 2\pi \frac{t}{T} \sin \Delta = \cos \left[2\pi \frac{t}{T} - \Delta \right].$$

This relation shows that the amplitude of vibration is E, if we compare it to $x = a \cos 2\pi \frac{t}{T}$.

We have:

$$E^2 (\cos^2 \Delta + \sin^2 \Delta) = E^2 = \begin{cases} a^2 (\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos 2\pi\varphi)^2 \\ + a^2 \sin^2 \alpha \sin^2 \beta \sin^2 2\pi\varphi. \end{cases}$$

$$\frac{E^2}{a^2} = \cos^2 \alpha \cos^2 \beta + \sin^2 \alpha \sin^2 \beta \cos^2 2\pi \varphi$$

$$+ 2 \cos \alpha \cos \beta \sin \alpha \sin \beta \cos 2\pi \varphi + \sin^2 \alpha \sin^2 \beta \sin^2 2\pi \varphi$$

$$= \cos^2 \alpha \cos^2 \beta + \sin^2 \alpha \sin^2 \beta + 2 \sin \alpha \sin \beta \cos \alpha \cos \beta$$

$$- 4 \sin \alpha \sin \beta \cos \alpha \cos \beta \sin^2 \pi \varphi.$$

Replacing $\cos 2\pi\varphi$ by $1 - 2 \sin^2 \pi \varphi$,

$$\frac{E^2}{a^2} = (\cos \alpha \cos \beta + \sin \alpha \sin \beta)^2 - \sin 2\alpha \sin 2\beta \sin^2 \pi \varphi$$

$$= \cos^2 (\alpha - \beta) - \sin 2\alpha \sin 2\beta \sin^2 \pi \varphi \quad . \quad . \quad . \quad . \quad . \quad (1)$$

DISCUSSION OF THE FORMULA.

We have that $\varphi = \frac{h}{\lambda \cos r} (n_e - n_o)$.

h represents the thickness of the section; r the angle between the rays, which we regard as superposed, and the normal to the faces of the section; n_e, n_o the indices of refraction of the two rays.

When white light, composed of an infinite number of lights of very different wave-lengths, is employed, the second term corresponds to the sum of the intensities of these lights, and the general formula may then be written:

$$\frac{E^2}{a^2} = \cos^2 (\alpha - \beta) - \sin 2\alpha \sin 2\beta \sum \sin^2 \pi \frac{h}{\lambda \cos r} (n_e - n_o) \quad . \quad . \quad (2)$$

THE DIFFERENT CASES.

I. $\alpha - \beta = \frac{\pi}{2}$, case where the principal sections of the polarizer and analyzer are at right angles to each other.

Since $\beta = \alpha - \frac{\pi}{2}$, formula (2) becomes:

$$\frac{E^2}{a^2} = \cos^2 \frac{\pi}{2} - \sin 2\alpha \sin (2\alpha - \pi) \sum \sin^2 \pi \varphi, \text{ or}$$

$$\frac{E^2}{a^2} = \sin^2 2\alpha \sum \sin^2 \pi \varphi = \sin^2 2\alpha \sum \sin^2 \left[\frac{\pi \Delta}{\lambda \cos r} \right] \quad . \quad . \quad . \quad (3)$$

Particular Cases.

1°. Let $\beta = 0$, then $\alpha = \frac{\pi}{2}$. The axes OX, OY of the plate are parallel to the principal sections of both analyzer and polarizer. All the colors are extinguished in the crystal at the same time, since they have the same axes of elasticity.

$$\frac{E^2}{a^2} = 0.$$

2°. $\beta = 45°$, $\alpha = \frac{\pi}{2} + 45°$.

$$\frac{E^2}{a^2} = \sum \sin^2 \pi \varphi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

II. $\alpha = \beta$, the polarizer and analyzer are parallel.

$$\frac{E^2}{a^2} = 1 - \sin^2 2\alpha \sum \sin^2 \pi \varphi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The values (3) and (5) are complementary.

2°. $\alpha = 45°$, $\beta = 45°$.

$$\frac{E^2}{a_2} = 1 - \sum \sin^2 \pi \varphi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

All the colors have their maximum of intensity.

If we employ a rhombohedron of calcite instead of a nicol, we have two images. The intensity $\frac{O^2}{a^2}$ of the ordinary image is obtained by means of formula (2) by replacing α by $\alpha + \frac{\pi}{2}$; the formula becomes:

$$\frac{O^2}{a^2} = \sin^2 \alpha \cos^2 \beta + \sin^2 \beta \cos^2 \alpha$$
$$- 2 \sin \alpha \sin \beta \cos \alpha \cos \beta \cos 2\pi \varphi$$
$$= \sin^2 (\alpha - \beta) + \sin 2\alpha \sin 2\beta \sum \sin^2 \pi \varphi. \quad . \quad . \quad (7)$$

These phenomena are complementary to those which give $\frac{E^2}{a^2}$.

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